

# Confidence Intervals for Proportions

## Lecture 31 Sections 9.4

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# Outline

## Confidence Intervals for Proportions

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# Estimation

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- We now turn to the other question about a population parameter.

## Question

What is the value of the parameter?

- This is called **estimation**.

# Point Estimates

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## Definition (Point Estimate)

A single number that estimates the value of the parameter.

- $\bar{x}$  is a point estimator for  $\mu$ .
- $\hat{p}$  is a point estimator for  $p$ .
- $s^2$  is a point estimator for  $\sigma^2$ .
- The problem with point estimates is that we have no idea how close we can expect them to be to the parameter.
- That is, we have no idea of how large the error may be.

# Interval Estimates

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## Definition (Interval Estimate)

An interval of numbers that has a specified probability of containing the value of the parameter.

- An interval estimate is more informative than a point estimate.
- Half the width of the interval is the **margin of error**.

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- $\bar{x} \pm$  margin of error is an interval estimate for  $\mu$ .
- $\hat{p} \pm$  margin of error is an interval estimate for  $p$ .
- $s^2 \pm$  margin of error is an interval estimate for  $\sigma^2$ .

## Question

How do we find the margin of error?

- It depends on the “level of confidence.”

# Confidence Levels

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### Definition (Confidence level)

The probability that the interval estimate contains the value of the parameter.

- If the confidence level is 95%, then the interval estimate is called a **95% confidence interval**.
- Treating the center of the interval estimate as the point estimate, we are 95% sure that the value of the parameter is within the margin of error from the point estimate.

# 95% Confidence Intervals

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- How do we find a 95% confidence interval for  $p$ ?
- Begin with the sample size  $n$  and the sampling distribution of  $\hat{p}$ .
- We know that the sampling distribution is normal with mean  $p$  and standard deviation

$$\sqrt{\frac{p(1-p)}{n}}.$$

- Therefore, 95% of the time,  $\hat{p}$  will lie within  $1.96\sqrt{\frac{p(1-p)}{n}}$  of  $p$ .

# The Target Analogy

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- Consider an analogy of a shooter shooting at a target.
- Suppose a shooter hits within 5 rings (5 inches) of the bull's eye 95% of the time.
- Then each individual shot has a 95% chance of hitting within 5 inches.

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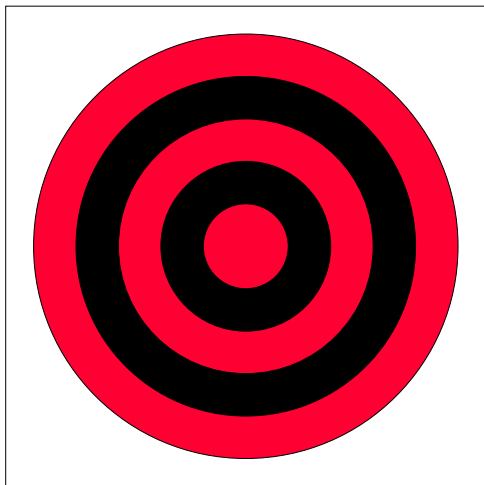
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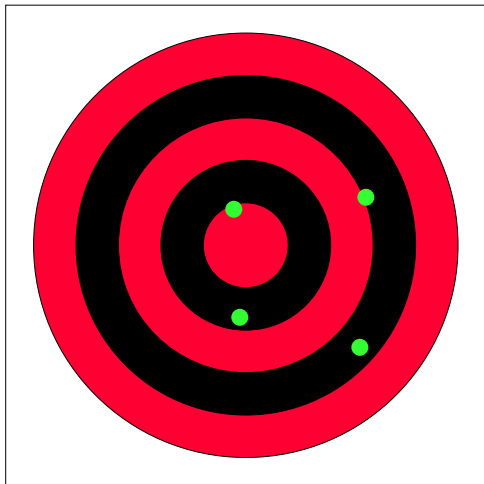
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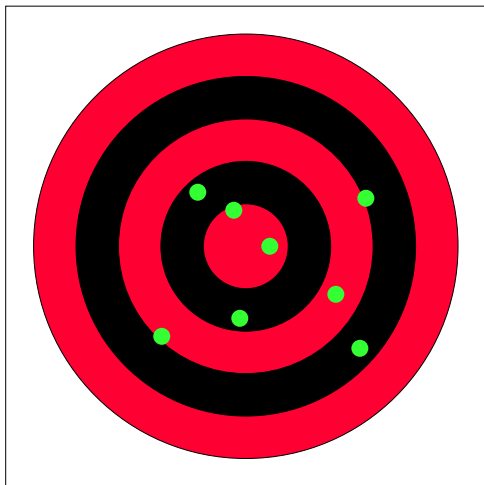
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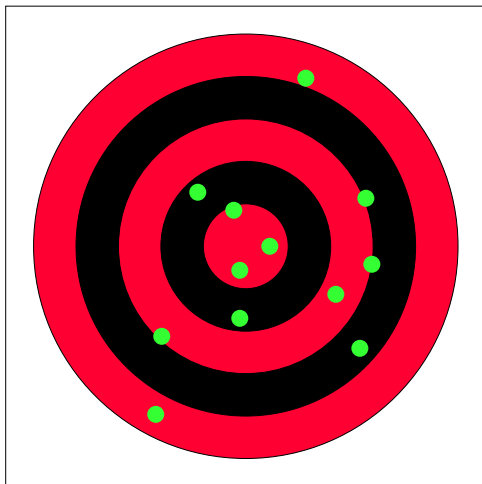
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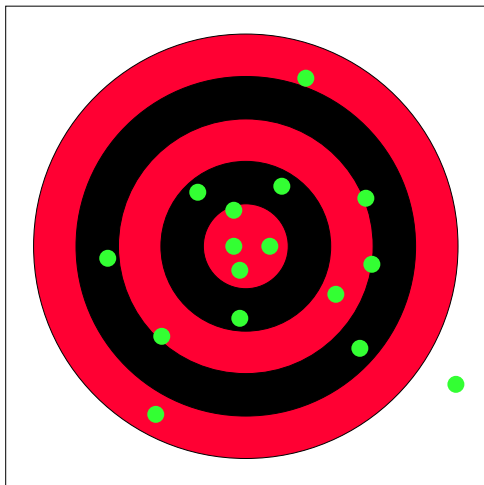
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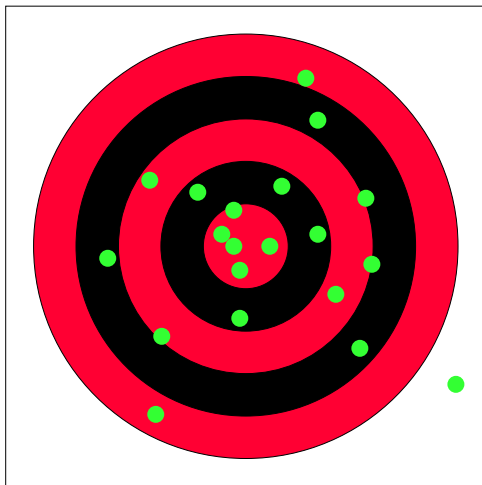
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- Now suppose we are shown where the shot hit, but we are not shown where the bull's eye is.
- What is the probability that the bull's eye is within 5 inches of that shot?

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Where is the bull's eye?

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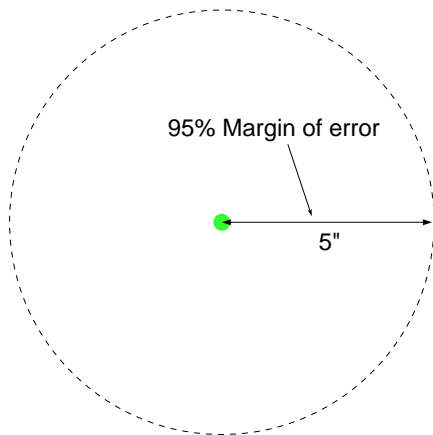
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# The Confidence Interval

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- In a similar way, 95% of the sample proportions  $\hat{p}$  should lie within 1.96 standard deviations ( $\sigma_{\hat{p}}$ ) of the parameter  $p$ .

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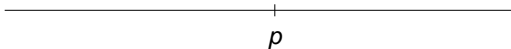
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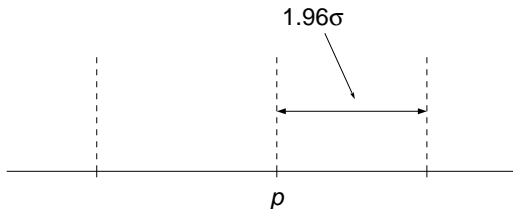
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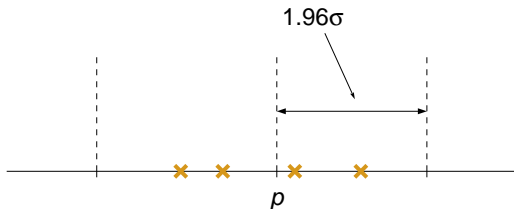
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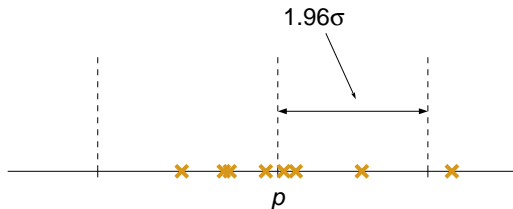
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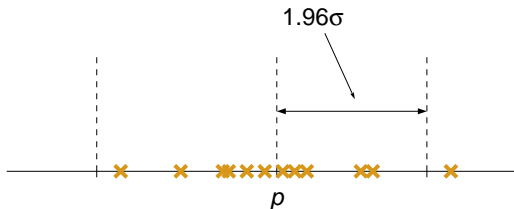
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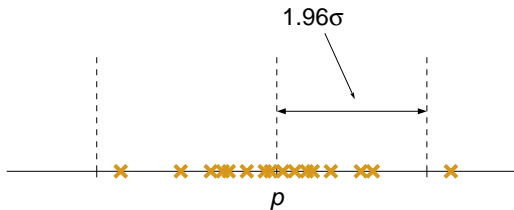
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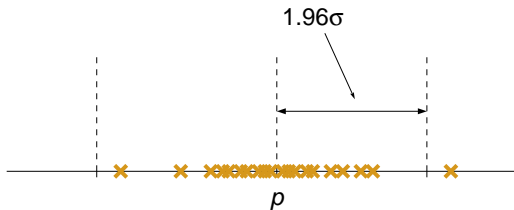
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- Therefore, if we compute a single  $\hat{p}$ , then there is a 95% chance that it lies within a distance  $1.96\sigma_{\hat{p}}$  of  $p$ .

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Where is  $p$ ?

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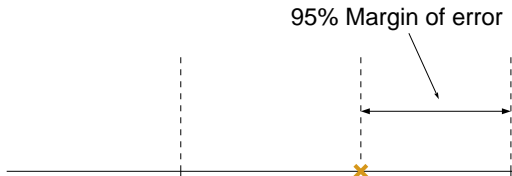
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# Approximate 95% Confidence Intervals

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- Thus, the 95% confidence interval *would* be

$$\hat{p} \pm 1.96\sigma_{\hat{p}}.$$

- The trouble is,

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.$$

- Therefore, to know  $\sigma_{\hat{p}}$ , we must know  $p$ , which is the very thing we are trying to estimate!

# Approximate 95% Confidence Intervals

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- The best we can do is to use  $\hat{p}$  in place of  $p$  to estimate  $\sigma_{\hat{p}}$ :

$$\hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$

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- Suppose we observe that of 1000 births, 520 of them are male.
- Estimate, using a 95% confidence interval, the true proportion of male births.

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- The point estimate is

$$\hat{p} = \frac{520}{1000} = 0.52.$$

- The margin of error is

$$1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{(0.52)(0.48)}{1000}} = 0.0310.$$

- The confidence interval is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.52 \pm 0.0310.$$

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$$\hat{p} = \frac{520}{1000} = 0.52.$$

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$$1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{(0.52)(0.48)}{1000}} = 0.0310.$$

- The confidence interval is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.52 \pm 0.0310.$$

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- The point estimate is

$$\hat{p} = \frac{520}{1000} = 0.52.$$

- The margin of error is

$$1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{(0.52)(0.48)}{1000}} = 0.0310.$$

- The confidence interval is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.52 \pm 0.0310.$$

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- We are 95% confident that the point estimate of 0.52 is in error by no more than 0.0310.

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- The point estimate is  $\hat{p}$ .
- The margin of error is half the width of the confidence interval, namely

$$1.96\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$

- The interval estimate centered at  $\hat{p}$  is the confidence interval

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$